Statistical Thermodynamics of Solutions of Optically Active Substances. III. Influence of an Optically Active Substance on the Equilibrium between Interconvertible d- and l-Optical Isomers

By Kazuo AMAYA

(Received June 1, 1962)

If an optically active substance is added to a mixture of d- and l-optical isomers which are interconvertible to each other, the original equilibrium between d- and l-isomers is expected to change. This seems to cause a new type of solvent effect on the optical rotation, the feature of which is that the solvent molecules themselves contribute to the optical rotation. Before the addition of an optically active substance there exist an equal number of the molecules of d- and l-isomers in the mixture, so it shows no optical rotation, but after that addition, the number of molecules of d- and l-isomers in equilibrium is expected to become unequal; this unbalanced number of molecules of either d- or l-isomer contribute to the total optical rotation.

This problem also seems to be important for studying asymmetric synthesis in an optically active solvent, because it is intimately related to the optical purity of the product.

In order to clarify the fundamental features of these problems, the statistical mechanics of such systems have been developed and the influence of the addition of optically active substances on the equilibrium between interconvertible d- and l-optical isomers has been examined.

Theory

Consider a system consisting of N_A molecules of either d- or l-isomer of an optically active substance of species A and of N_B molecules of a mixture of interconvertible d- and l-isomers of species B, namely, $N_B = N_{B_d} + N_{B_l}$ where N_{B_d} and N_{B_l} denote the number of molecules of d- and l-isomers respectively.

For the sake of simplicity, assumptions similar to those described in previous papers[1] are made. These are briefly summarized here again:

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[1] K. Amaya, This Bulletin, **34**, 1689, 1803 (1961).

1) The molecules of species A and B are covered by z planes. One of them consists of 3 different subelements; the arrangement of these three subelements for d- and l-isomers are mirror images of one another (this plane will be called the optically active plane). The remaining z-1 planes, composed of one uniform element and the pair of optically active planes in contact, can take three sets of configuration of a different energies.

2) The molecules of A and B have the same molecular volume; these molecules occupy the lattice points of the quasi-crystalline lattice of the nearest neighbors of z, and the molecule at a given site posseses $3z$ orientational freedom.

3) The configurational energy of the system is the sum of the potential energy of the $1/2 \cdot z(N_A+N_B)$ pairs of the surface plane of the molecules in contact.

To make the fundamental features of the problem as clear as possible, mathematical rigorousness is abandoned and we now introduce a concept of effective potential energy for a pair of optically active planes and attribute any difference between two optical isomers to it. Here "effective" means that we take the weighted mean of the potential energy of a given pair of planes in contact by averaging three different configurations by multiplying each Boltzmann factor.

Following the above assumptions, it can easily be expected that the non-zero value of the effective interchange energy of B_d and B_l molecules, w_{dl}, arises only from the pair of optically active planes, and that the differences in the effective interchange energy of A and B_d molecules, w_{AB_d}, and that of A and B_l, w_{AB_l}, also arise from the difference in the potential energy of the pairs of optically active planes of the A and B_d molecules and those of the A and B_l molecules.

Following the model described above, $w_{AB_d}-\overline{w}_{AB_l}$ and w_{dl} will be estimated. If the potential energy of the i-th configuration of a pair of optically active planes from the pairs of molecules A–A, B_d–B_d, B_l–B_l, A–B_d, A–B_l and B_d–B_l are denoted by $U^{aa}_{AA_i}$, $U^{dd}_{BB_i}$, $U^{ll}_{BB_i}$, $U^{ad}_{AB_i}$, $U^{al}_{AB_i}$ and $U^{ld}_{BB_i}$ respectively, the weighted mean potential energy of each pair over three different configurations may be represented as:

$$\overline{U}^{aa}_{AA}=\sum_{i=1}^{3} U^{aa}_{AA_i}\,e^{-U^{aa}_{AA_i}/kT}\Big/\sum_{i=1}^{3} e^{-U^{aa}_{AA_i}/kT}$$

$$U^{dd}_{BB}=\sum_{i=1}^{3} U^{dd}_{BB_i}\,e^{-U^{dd}_{BB_i}/kT}\Big/\sum_{i=1}^{3} e^{-U^{dd}_{BB_i}/kT}$$

$$U^{ll}_{BB}=\sum_{i=1}^{3} U^{ll}_{BB_i}\,e^{-U^{ll}_{BB_i}/kT}\Big/\sum_{i=1}^{3} e^{-U^{ll}_{BB_i}/kT}$$

$$\overline{U}^{ab}_{AB}=\sum_{i=1}^{3} U^{ad}_{AB_i}\,e^{-U^{ad}_{AB_i}/kT}\Big/\sum_{i=1}^{3} e^{-U^{ad}_{AB_i}/kT}$$

$$\overline{U}^{al}_{AB}=\sum_{i=1}^{3} U^{al}_{AB_i}\,e^{-U^{al}_{AB_i}/kT}\Big/\sum_{i=1}^{3} e^{-U^{al}_{AB_i}/kT}$$

$$\overline{U}^{dl}_{AB}=\sum_{i=1}^{3} U^{dl}_{AB_i}\,e^{-U^{dl}_{AB_i}/kT}\Big/\sum_{i=1}^{3} e^{-U^{dl}_{AB_i}/kT}$$

Since each term of $U^{ad}_{AB_i}$ has a different value, from the corresponding term in $U^{al}_{AB_i}$, the effective potential energies, $\overline{U}^{ad}_{AB}$ and $\overline{U}^{al}_{AB}$, have different values. In a similar way, it is deduced that $\overline{U}^{dl}_{BB}$ and U^{dd}_{BB} or $\overline{U}^{dl}_{BB}$ and $\overline{U}^{ll}_{BB}$ have different values. In this way, d and l optical isomers in the system can be made distinguishable energetically.

Further, we define the following quantity:

$$\chi_A=\frac{1}{2z}\left\{(z-1)^2U^{bb}_{AA}+2(z-1)U^{ab}_{AA}+U^{aa}_{AA}\right\}$$

$$\chi_{B_d}=\frac{1}{2z}\left\{(z-1)^2U^{bb}_{BB}+2(z-1)U^{ab}_{BB}+\overline{U}^{dd}_{BB}\right\}$$

and

$$\chi_{B_l}=\frac{1}{2z}\left\{(z-1)^2U^{bb}_{BB}+2(z-1)U^{ab}_{BB}+U^{ll}_{BB}\right\}$$

where χ_A, χ_{B_d}* and χ_{B_l}* indicate the mean potential energy per molecule of the A, B_b and B_l molecules respectively; U^{bb}_{AA} and U^{bb}_{BB} are the potential energies of a pair of planes which are not optically active of the A and B molecules respectively, and U^{ab}_{AA} and U^{ab}_{BB} are those of a pair of** planes, one of which is optically active and the other of which is optically inactive for A and B molecules respectively.

The interchange energy of A and B_d, w_{AB_d}, that of A and B_l, $\overline{w}_{AB_l}$, and that of B_d and B_l, $\overline{w}_{dl}$, molecules are, by definition, expressed as follows:

$$2\overline{w}_{AB_d}=\frac{2}{z}\Big\{(z-1)^2U^{bb}_{AB}+(z-1)\times(U^{ab}_{AB}+U^{ba}_{AB})+\overline{U}^{ad}_{AB}\Big\}-2\chi_A-2\chi_{B_d}$$

$$2\overline{w}_{AB_l}=\frac{2}{z}\Big\{(z-1)^2U^{bb}_{AB}+(z-1)\times(U^{ab}_{AB}+U^{ba}_{AB})+\overline{U}^{al}_{AB}\Big\}-2\chi_A-2\chi_{B_l}$$

$$2\overline{w}_{dl}=\frac{2}{z}\Big\{(z-1)^2U^{bb}_{AB}+2(z-1)+U^{ab}_{BB}+\overline{U}^{dl}_{BB}\Big\}-2\chi_{B_d}-2\chi_{B_l}$$

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* χ_{B_d} and χ_{B_l} are imaginary because B_d and B_l are interconvertible and an assembly of pure B_d or B_l molecule cannot exist.

** The suffix "a" denotes optically active and "b" denotes optically inactive.

Since $\bar{\bar{U}}_{\mathrm{BB}}^{\mathrm{dd}}=\bar{\bar{U}}_{\mathrm{BB}}^{\mathrm{ll}}$ $(=\bar{\bar{U}}_{\mathrm{BB}}^{\mathrm{aa}})$ and, hence, $\chi_{\mathrm{B_d}}=\chi_{\mathrm{B_l}}$ $(=\chi_{\mathrm{B}})$, it follows that

$$\Delta\bar{\bar{w}}=\bar{\bar{w}}_{\mathrm{AB_d}}-\bar{\bar{w}}_{\mathrm{AB_l}}=\frac{1}{z}(\bar{\bar{U}}_{\mathrm{AB}}^{\mathrm{ad}}-\bar{\bar{U}}_{\mathrm{AB}}^{\mathrm{al}})$$

$$\bar{\bar{w}}_{\mathrm{dl}}=\frac{1}{z}(\bar{\bar{U}}_{\mathrm{BB}}^{\mathrm{dl}}-\bar{\bar{U}}_{\mathrm{BB}}^{\mathrm{aa}})$$

Having thus defined these quantities, we can proceed to the next step.

Before considering the ternary system, we must first consider the pure solvent system, a binary system composed of $N_{\mathrm{B_d}}$ molecules of $\mathrm{B_d}$ and $N_{\mathrm{B_l}}$ molecules of $\mathrm{B_l}$.

The configurational free energy of the system $F_{\mathrm{C}}{}^{\mathrm{B}}$ is represented as:

$$F_{\mathrm{C}}{}^{\mathrm{B}}=-kT\ln\frac{(N_{\mathrm{B_d}}+N_{\mathrm{B_l}})!}{N_{\mathrm{B_d}}!\,N_{\mathrm{B_l}}!}+\frac{N_{\mathrm{B_d}}\,N_{\mathrm{B_l}}}{N_{\mathrm{B}}{}^2}\,\bar{\bar{w}}_{\mathrm{dl}}$$
$$-\chi_{\mathrm{B}}\,N_{\mathrm{B}}-kT\,N_{\mathrm{B}}\ln 3\,z$$

In the equilibrium state, $F_{\mathrm{C}}{}^{\mathrm{B}}$ must be the minimum.

Then, the first derivative of $F_{\mathrm{C}}{}^{\mathrm{B}}$ with respect to $N_{\mathrm{B_d}}$ under the condition that $N_{\mathrm{B}}=N_{\mathrm{B_d}}+N_{\mathrm{B_l}}$ $=$constant must be zero, we have

$$\left(\frac{\delta F_{\mathrm{C}}{}^{\mathrm{B}}}{\delta N_{\mathrm{B_d}}}\right)_{N_{\mathrm{B}}}=kT\ln N_{\mathrm{B_d}}/(N_{\mathrm{B}}-N_{\mathrm{B_d}})$$
$$+(N_{\mathrm{B}}-2N_{\mathrm{B_d}})w_{\mathrm{dl}}=0$$

Since the $\mathrm{B_d}$ and $\mathrm{B_l}$ molecules are equivalent, the same relation must hold for $\mathrm{N_{B_l}}$. Therefore, we obtain, by using the relation $N_{\mathrm{B_l}}=N_{\mathrm{B}}-N_{\mathrm{B_d}}$;

$$\left(\frac{\delta F_{\mathrm{C}}{}^{\mathrm{B}}}{\delta N_{\mathrm{B_l}}}\right)_{N_{\mathrm{B}}}=kT\ln(N_{\mathrm{B}}-N_{\mathrm{B_d}})/N_{\mathrm{B_d}}$$
$$+(2\,N_{\mathrm{B_d}}-N_{\mathrm{B}})\bar{\bar{w}}_{\mathrm{dl}}=0$$

For the simultaneous satisfaction of these two relations both terms must always be zero; it follows that

$$N_{\mathrm{B_d}}=N_{\mathrm{B_l}}=\frac{1}{2}N_{\mathrm{B}}$$

Next, we consider the solution, a ternary system composed of A, $\mathrm{B_d}$ and $\mathrm{B_l}$ molecules.

The configurational free energy of the system F_{C} is represented by

$$F_{\mathrm{C}}=-N_{\mathrm{A}}\chi_{\mathrm{A}}-N_{\mathrm{B}}\chi_{\mathrm{B}}-kT\ln\frac{(N_{\mathrm{A}}+N_{\mathrm{B}})!}{N_{\mathrm{A}}!\,N_{\mathrm{B_d}}!\,N_{\mathrm{B_l}}!}$$
$$+\frac{N_{\mathrm{A}}\,N_{\mathrm{B_d}}}{N_{\mathrm{A}}+N_{\mathrm{B}}}\bar{\bar{w}}_{\mathrm{AB_d}}+\frac{N_{\mathrm{A}}\,N_{\mathrm{B_l}}}{N_{\mathrm{A}}+N_{\mathrm{B}}}(\bar{\bar{w}}_{\mathrm{AB_d}}-\Delta\bar{\bar{w}})$$
$$+\frac{N_{\mathrm{B_d}}\,N_{\mathrm{B_l}}}{(N_{\mathrm{A}}+N_{\mathrm{B}})}\,\bar{\bar{w}}_{\mathrm{dl}}-(N_{\mathrm{A}}+N_{\mathrm{B}})kT\ln 3z$$

The equilibrium concentration of $\mathrm{B_d}$ or $\mathrm{B_l}$ molecules is the one that minimizes F_{C}. This can be obtained by differentiating F_{C} with respect to $N_{\mathrm{B_d}}$ or $N_{\mathrm{B_l}}$, under the condition that $N_{\mathrm{B}}=N_{\mathrm{B_d}}+N_{\mathrm{B_l}}$ is constant, and by then equating it with zero. Thus, we obtain

$$\delta F_{\mathrm{C}}/\delta N_{\mathrm{B_d}}=Tk\ln\frac{N_{\mathrm{B_d}}}{N_{\mathrm{B}}-N_{\mathrm{B_d}}}+N_{\mathrm{A}}/(N_{\mathrm{A}}+N_{\mathrm{B}})\Delta w$$
$$+(N_{\mathrm{B}}-2\,N_{\mathrm{B_d}})/(N_{\mathrm{A}}+N_{\mathrm{B}})w_{\mathrm{dl}}=0$$

By introducing the notations y and x_{A} defined by the following equations:

$$y=\frac{N_{\mathrm{B_d}}}{N_{\mathrm{B}}},\qquad x_{\mathrm{A}}=N_{\mathrm{A}}/(N_{\mathrm{A}}+N_{\mathrm{B}})$$

the above equation may be rewritten as:

$$kT\ln y/(1-y)+x_{\mathrm{A}}\,\Delta w-(1-x_{\mathrm{A}})(1-2y)w_{\mathrm{dl}}$$
$$=0$$

When y is close to 0.5, y can be replaced by $0.5+\Delta$. The logarithmic term is expanded in the power series in Δ and, by neglecting terms higher than the second, we can obtain the following approximate solution for Δ:

$$\Delta=-\frac{x_{\mathrm{A}}\cdot\Delta w}{4\,kT-(1-x_{\mathrm{A}})w_{\mathrm{dl}}}$$

Discussion

Since the value of Δ is usually different from zero, the equilibrium is shifted in either direction. If the sign of Δ is positive, the D isomers increase, and vice versa.

The absolute value of Δ increases as x_{A} increases, and the maximum value of $|\Delta|$ is $|\Delta\bar{w}/4\,kT|$. Usually $w_{\mathrm{dl}}\ll kT$, and $\bar{w}_{\mathrm{dl}}$ can be neglected as too small, and the $|\Delta|$ is approximated to $\Delta\bar{w}/4\,kT\cdot x_{\mathrm{A}}$. Since $\Delta\bar{w}$ is nearly proportional to $1/T$, $|\Delta|$ is nearly proportional to $1/T^2$. The extent of the shift from the original equilibrium state become greater as the temperature decreases. This indicates that, in order to get optically active polymers of a large optical rotatory power by the polymerization of monomers in an optically active solvent, it is desirable to perform polymerization at as low temperatures and as low concentrations (large value of x_{A}) as possible.

In order to get direct evidence of this predicted phenomena, it seems to be most suitable to measure the optical rotatory dispersion of these mixtures, since, by the measurement of optical rotation with the light of a single wavelength, it is difficult to distinguish the contributions of the solute from that of the solvent in the observed optical rotation.

However, if the optical rotatory dispersion is measured, the contributions from each species can easily be distinguished, for usually the dispersion curve is different and the maxima of optical rotation appear at different wavelengths. Humps may be observed on the optical rotatory dispersion curve of the solute. (No such hump is expected for optically inactive solvents.) Another indirect evidence may be obtained by the study of polymerization in an optically active medium. This kind of study has already been done²⁾, but the effect of the temperature of polymerization is not apparent.

Government Chemical Industrial
Research Institute
Shibuya-ku, Tokyo

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2) G. Natta, M. Farina, M. Peraldo and G. Bressan, *Makromol. Chem.*, **43**, 68 (1961).